Compounding Math

P1. Krish makes one time investment of $x = \$1{,}000$ dollars that compounds annually at r = 3% interest rate.

P1a. What is the future value of the investment $FV(t, r, x)$ after one year?

P1b. What is the future value $FV(t, r, x)$ of the investment after two years?

P1c. What is the future value $FV(t, r, x)$ of the investment after three years?

P1d. What is the future value $FV(t, r, x)$ of the investment after ***t*** years?

P2. Sophie makes onetime investment of $x = \$1{,}000$ dollars that compounds monthly (i.e. n = 12 compounding periods in a year) at r = 3% annual interest rate.

P2a. What is the future value of the investment $FV(t, r, n, x)$ after one year?

P2b. What is the future value $FV(t, r, n, x)$ of the investment after two years?

P2c. What is the future value $FV(t, r, n, x)$ of the investment after three years?

P2d. What is the future value $FV(t, r, n, x)$ of the investment after t years assuming n compounding periods per year?

Math for Future Value Annuities

An annuity is a series of payments made at equal intervals, a.k.a. periods. In the ordinary annuity the payment is made at the and of the period. In the annuity due the payment is made at the beginning of the period.

P3. Krish has made a new year resolution to deposit $\$1{,}000$ dollars to a savings account on 31 December for the next 4 years. The savings account pays 3% interest rate compounded annually. Krish made the first deposit on 31 December 2019. What is the future value of the savings after the fourth deposit on 31 December 2022?

P3a. Solve the problem by calculating the future value of each deposit separately.

The future value of x dollars compounded over t periods, i.e. years in the current example, and using r interest rate per period is calculated as follows

$$FV(t, r, x) = (1 + r)^t x$$

Using the above formula calculate the future value of each deposit.

P3b. Sum up all the future values of deposits to get the future value of the savings on 31 December 2022

P4. Krish has made a new year resolution to deposit $\$1{,}000$ dollars to a savings account on 31 December for the next 4 years. The savings account pays 3% interest rate compounded annually. Krish made the first deposit on 31 December 2019. What is the future value of the savings after the fourth deposit on 31 December 2022?

P4a. Solve the problem by using formula for ordinary annuity (a.k.a. annuity in arrears).

P4b. Compare the calculated future value of ordinary annuity to the sum of future values of all four deposits calculated in problem 3).

P4c. Derive the formula for ordinary annuity.

P5. Krish is wondering how much the future value of the savings would be worth on 31 December 2022 if deposits of $\$1{,}000$ dollars to a savings account were made on the 1st January for 4 years starting in 2019. The savings account pays 3% interest rate compounded annually.

P5a. Solve the problem by using formula for annuity "due" (a.k.a. annuity in advance).

P5b. Derive formula for annuity due.

P6. Krish has made a new year resolution to make deposits to a savings account on 31 December for the next 4 years. The savings account pays 3% interest rate compounded annually. Krish made the first deposit of $1,000 on 31 December 2019. Krish plans to increase the amount of deposit by 20% each year.

P6a. What are the deposits for 2019, 2020, 2021 and 2022?

P6b. What will be the future value of each deposit on 31 December 2022?

P6c. What will be the future value of the savings after the fourth deposit on 31 December 2022?

P7. Krish has made a new year resolution to make deposits to a savings account on 31 December for the next 4 years. The savings account pays 3% interest rate compounded annually. Krish made the first deposit of $1,000 on 31 December 2019. Krish plans to increase the amount of deposit by 20% each year.

P7a. Use the future value growth annuity formula to get the future value of savings on 31 December 2022.

P7b. Compare the future value of the ordinary growth annuity to the sum of future values of all four deposits calculated in problem 6.

P7c. Derive the future value growth annuity formula.

P8. Martha is saving for a new car in her TFSA account which earns 3% annualized interest compounded monthly. At the end of each month, she deposits $500.00 into her account. How much will she have in the account after 5 years?

P9. Kathy is saving for her retirement and opens an RRSP - which earns 2.5% annualized interest compounded monthly. At the end of each week, she deposits $100.00 into her account. How much will she have in the account after 35 years?

P9a. Calculate equivalent weekly rate.

P9b. Calculate the future value of the annuity

P10. Anton uses his investment account to purchase $600 worth of mutual funds. Every two months, he purchases an additional $200 worth of fund units. How much will he have in his account after 10 years, assuming the mutual fund pays 5%, compounded semi-annually.

P10a. Calculate the future value of the initial $600 mutual fund investment

P10b. Calculate equivalent 2-month rate

P10c. Calculate the future value of the ordinary annuity

P10d. Calculate the future value of the mutual fund holdings

P11. August is saving for a down-payment on a home. She has a TFSA with nothing in it and plans to make regular monthly deposits at the end of each month. After 6 years, she has $5,000 in the account. If her TFSA has a 4% interest rate compounded quarterly, how much did she deposit each month?

P11a. Calculate equivalent monthly rate.

P11b. Calculate monthly deposits.

Math for Present Value Annuities

P12. Jackie has bank student loans, at 3% interest compounded monthly. She is making payments at the end of every four months, $200.00 each and will have paid the loan back within 5 years.

P12a. What is the equivalent rate for a 4-month period?

P12b. How much did she borrow?

P12c. How much total interest did she pay?

P13. Arnold wants to open a restaurant and decides he needs to borrow $80,000. He sells his baseball card collection for $15,500 and gives the money to the bank as a deposit. In return, they offer to lend him the rest at 6% interest, compounded quarterly. The bank gives Arnold 9 years to repay the loan. The payments are due on the **first** day of each month.

P13a. What is the equivalent rate for 4-month period?

P13b. What is his monthly payment?

P13c. How much total interest did he pay?

P14. Arnold wants to open a restaurant and decides he needs to borrow $80,000. He sells his baseball card collection for $15,500 and gives the money to the bank as a deposit. In return, they offer to lend him the rest at 6% interest, compounded quarterly. The bank gives Arnold 9 years to repay the loan. The payments are due on the **last** day of each month.

P14a. What is the equivalent rate for 4-month period?

P14b. What is his monthly payment?

P14c. How much total interest did he pay?

Activity Answer Key

A1a.

$FV \rightarrow FV(t=1, r=3\%, x=\$1{,}000) = (1+r)x$ = $(1+0.03)$* \$1,000 = \$1,030

A1b.

$FV \circ FV \rightarrow FV(t=2, r=3\%, x=\$1{,}000) = FV\big(1, r, FV(1,x,r)\big) = (1+0.03)*(1+0.03)*$ $\$1{,}000 =$ 1.0609 * \$1,000 = \$1,060.90

A1c.

$FV \circ FV \circ FV \rightarrow FV(t=3, r=3\%, x=\$1{,}000) = FV\big(1, r, FV(2,x,r)\big) = (1+0.03)*$ $(1+0.03)*(1+0.03)*\$1{,}000 =$ 1.09273 * \$1,000 = \$1,092.73

A1d.

$FV(t,r,x) = (1+r)^t x$

A2a.

$FV \rightarrow FV(t=1, r=3\%, n=12,\ x=\$1{,}000) = \left(1+\frac{0.03}{12}\right)^{12}$* \$1,000 = \$1,030.42

A2b.

$FV \circ FV \rightarrow FV(t=2, r=3\%, n=12, x=\$1{,}000) = FV\big(1, r, n, FV(1,r,n,x)\big) = \left(1+\frac{0.03}{12}\right)^{12}\left(1+\frac{0.03}{12}\right)^{12} * \$1{,}000 = \left(1+\frac{0.03}{12}\right)^{12*2} * \$1{,}000 =$ 1.06176 * \$1,000 = \$1,061.76

A2c.

$FV \circ FV \rightarrow FV(t=3, r=3\%, n=12, x=\$1{,}000) = FV\big(1, r, n, FV(2,r,n,x)\big) = \left(1+\frac{0.03}{12}\right)^{12}\left(1+\frac{0.03}{12}\right)^{12*2} * \$1{,}000 = \left(1+\frac{0.03}{12}\right)^{12*3} * \$1{,}000 =$ 1.09405 * \$1,000 = \$1,094.05

A2d.

$FV \circ FV \circ FV \rightarrow FV(t,r,n,x) = \left(1+\frac{r}{n}\right)^{nt} x$

A3a.

1st deposit: What is the future value $FV(t,r,x)$ on 31 December 2022 of the 1st deposit made on 31 December 2019 after three years of compounding?
2019: $FV(t=3, r=3\%, x=\$1{,}000) = (1+0.03)^3 * \$1{,}000 =$ $1,092.73

Note that there I no compounding in 2019 as deposit was made on 31 December 2019. Thus, the deposit is compounded through 2020, 2021 and 2020, i.e. through 3 years

2nd deposit: What is the future value $FV(t,r,x)$ on 31 December 2022 of the 2nd deposit made on 31 December 2020 after two years of compounding?
2020: $FV(t=2, r=3\%, x=\$1{,}000) = (1+0.03)^2 * \$1{,}000 =$ $1,060.90

3rd deposit: What is the future value $FV(t,r,x)$ on 31 December 2022 of the 3rd deposit made on 31 December 2021 after one year of compounding?
2021: $FV(t=1, r=3\%, x=\$1{,}000) = (1+0.03)^1 * \$1{,}000$ = $1,030

4th deposit: What is the future value $FV(t,r,x)$ on 31 December 2022 of the 4th deposit made on 31 December 2022?
2022: $FV(t=0, r=3\%, x=\$1{,}000) = (1+0.03)^0 * \$1{,}000$ = $1,000

A3b. Add up future values of all four deposits to calculate the future value of the savings on 31 December 2022, namely:

$$Sum = [(1+r)^3 + (1+r)^2 + (1+r)^1 + (1+r)^0\,]\,x$$
$$Sum = [(1+0.03)^3 + (1+0.03)^2 + (1+0.03)^1 + (1+0.03)^0\,] * \$1{,}000$$
$$Sum = \$1{,}092.73 + \$1{,}060.90 + \$1{,}030 + \$1{,}000 = \$4{,}183.63$$

A4a. Use formula to calculate future value of ordinary annuity

$$Annuity\ FV(t,r,x) = \frac{-1+(1+r)^t}{r}x$$
$$Annuity\ FV(t=4, r=3\%, x=\$1{,}000) = \frac{-1+(1+0.03)^4}{0.03} * \$1{,}000 = \$4{,}183.63$$

Note that t stands for number of years that annuity covers, i.e. from 2019 to 2022, inclusive.

A4b. The future value of the ordinary annuity and the sum of future values of all annuity payments (i.e. deposits made to saving account in problem 3) are the same. An annuity is a series of payments made at equal intervals, a.k.a. periods. In the ordinary annuity the payment is made at the and of the period. The four deposits made on 31 December over the four consecutive years constitute an ordinary annuity.

Thus, the future value of the ordinary annuity on 31 December 2022 can be calculated by summing up future values of all four deposits.

$$Annuity\ FV(t,r,x) = [(1+r)^{t-1} + (1+r)^{t-2} + (1+r)^{t-3} + \cdots + (1+r)^{t-t}\,]\,x$$
$$Annuity\ FV(t=4, r=3\%, x=\$1{,}000) = [(1+0.03)^3 + (1+0.03)^2 + (1+0.03)^1 + (1+0.03)^0\,] * \$1{,}000$$
$$Annuity\ FV(t=4, r=3\%, x=\$1{,}000) = \$1{,}092.73 + \$1{,}060.90 + \$1{,}030 + \$1{,}000 = \$4{,}183.63$$

Note that t denotes number of annuity periods. The annuity covers 4 years: 2019, 2020, 2021 and 2022. For the ordinary annuity there is no compounding in the first period, i.e. in 2019, as the payment was made on 31 December 2019. Thus, there are only three compounding periods: 2020, 2021 and 2022.

A4c. The formula for an ordinary annuity is derived as follows:

Represent the ordinary annuity as the sum of future values of the deposits (a.k.a. annuity payments) as illustrated in an earlier problem

$$Annuity\ FV(t,r,x) = [(1+r)^0 + (1+r)^1 + (1+r)^2 + \cdots + (1+r)^{t-1}\,]\,x$$
(i)

Multiply both sides of the above equation (i) by (1+r)

$$Annuity\ FV(t,r,x) * (1+\mathrm{r}) = [(1+r)^1 + (1+r)^2 + (1+r)^3 + \cdots + (1+r)^t\,]\,x$$
(ii)

Subtract equation (i) from equation (ii), and simplify

$$Annuity\ FV(t,r,x) * (1+\mathrm{r}) - Annuity\ FV(t,r,x) = [-(1+r)^0 + (1+r)^t\,]\,x$$
$$Annuity\ FV(t,r,x) * \mathrm{r} = [-(1+r)^0 + (1+r)^t\,]\,x$$
$$Annuity\ FV(t,r,x) * \mathrm{r} = [-1 + (1+r)^t\,]x$$
(iii)

Divide both sides of equation (iii) by r

$$Annuity\ FV(t,r,x) = \frac{-1+(1+r)^t}{r}x$$

A5a. Use formula annuity due to calculate future value of the savings on 31 December 2022

$$Annuity\ Due\ FV(t,r,x) = (1+\mathrm{r}) * \frac{-1+(1+r)^t}{r}x$$
$$Annuity\ Due\ FV(t=4, r=3\%, x=\$1{,}000) = (1+0.03) * \frac{-1+(1+0.03)^4}{0.03} * \$1{,}000 = \$4{,}309.14$$

A5b. To derive formula for annuity due notice that each payment made at the beginning of the year compounds for one extra year as compared to the ordinary annuity, thus:

$$Annuity\ Due\ FV(t,r,x) = (1+r) * Annuity\ FV(t,r,x)$$
$$Annuity\ Due\ FV(t,r,x) = (1+\mathrm{r}) * \frac{-1+(1+r)^t}{r}x$$

A6a.

The annual deposits are made over four years (i.e. year = 1,2,3,4). The first deposit, x, is made at the end of 2019 (i.e. year = 1) and the subsequent deposits increase by 20% annually (i.e. growth rate g = 20%), namely:

$$Deposit\ (year, g, x) = (1+g)^{yea}$$

1st deposit made in year 1, i.e. in 2019

2019: $Deposit\ (year = 1, g = 20\%, x = \$1{,}000) = (1+0.2)^{1-1} * \$1{,}000 = \ \$1{,}000$

2nd deposit made in year 2, i.e. in 2020

2020: $Deposit\ (year = 2, g = 20\%, x = \$1{,}000) = (1+0.2)^{2-1} * \$1{,}000 = \ \$1{,}200$

3rd deposit made in year 3, i.e. in 2021

2021: $Deposit\ (year = 3, g = 20\%, x = \$1{,}000) = (1+0.2)^{3-1} * \$1{,}000 = \ \$1{,}440$

4th deposit made in year = 4, i.e. in 2022

2022: $Deposit\ (year = 4, g = 20\%, x = \$1{,}000) = (1+0.2)^{4-1} * \$1{,}000 = \ \$1{,}728$

A6b. The future value of each deposit on 31 December 2022 is calculated as follows;

$$FV\big(t, r, Deposit(year, g, x)\big) = \ (1+r)^t\ (1+g)^{year-1}\ x$$

The deposits are made over 4 years (i.e. T = 4). The table below illustrates that:

$$year = \ T - t$$

Thus, the future value of each deposit is calculated as

$$FV\big(t, r, Deposit(T-t, g, x)\big) = \ (1+r)^t\ (1+g)^{T-t-1}\ x$$

Deposit Date	year	t (number of years a deposit will compound)	year = T - t
31 December 2019	1	3	1 = 4 - 3
31 December 2020	2	2	2 = 4 - 2
31 December 2021	3	1	3 = 4 - 1
31 December 2022	4	0	4 = 4 - 0

1st deposit made in year 1, i.e. in 2019, will yield the following future value on 31 December 2022

2019: $FV\big(t = 3, r = 3\%, Deposit\ (year = 4 - 3, g = 20\%, x = \$1{,}000)\big) =$
$(1+0.03)^3\ (1+0.2)^{4-3-1} * \$1{,}000 = (1+0.03)^3 * \$1{,}000 = \$1{,}092.73$

2nd deposit made in year 2, i.e. in 2020, will yield the following future value on 31 December 2022
2020: $FV\big(t = 2, r = 3\%, Deposit\ (year = 4 - 2, g = 20\%, x = \$1{,}000)\big) = (1 + 0.03)^2\ (1 + 0.2)^{4-2-1} * \$1{,}000 = (1 + 0.03)^2 (1 + 0.2)^1 * \$1{,}000 = (1 + 0.03)^2 * \$1{,}200 = \$1{,}273.08$

3rd deposit made in year 3, i.e. in 2021, will yield the following future value on 31 December 2022
2021: $FV\big(t = 1, r = 3\%, Deposit\ (year = 4 - 1, g = 20\%, x = \$1{,}000)\big) = (1 + 0.03)^1\ (1 + 0.2)^{4-1-1} * \$1{,}000 = (1 + 0.03)^1 (1 + 0.2)^2 * \$1{,}000 = (1 + 0.03) * \$1{,}440 = \$1{,}483.20$

4th deposit made in year 3, i.e. in 2021, will yield the following future value on 31 December 2022
2022: $FV\big(t = 0, r = 3\%, Deposit\ (year = 4 - 0, g = 20\%, x = \$1{,}000)\big) = (1 + 0.03)^0\ (1 + 0.2)^{4-0-1} * \$1{,}000 = (1 + 0.2)^3 * \$1{,}000 = \$1{,}728.00$

A6c. The future value of the savings after the fourth deposit on 31 December 2022 is the sum of the future values of all four deposits:

$$Sum = (1 + r)^3\ (1 + g)^0 x + (1 + r)^2\ (1 + g)^1 x + (1 + r)^1\ (1 + g)^2 x + (1 + r)^0\ (1 + g)^3 x$$

$$Sum = (1 + 0.03)^3\ (1 + 0.2)^0 * \$1{,}000 + (1 + 0.03)^2\ (1 + g)^1 * \$1{,}000 + (1 + 0.03)^1\ (1 + 0.2)^2 * \$1{,}000 + (1 + r)^0\ (1 + g)^3 * \$1{,}000$$

$$Sum = \$1{,}092.73 + \$1{,}273.08 + \$1{,}483.20 + \$1{,}728 = \$5{,}577.01$$

A7a. The following are compact formulas for the future value of the ordinary growth annuity and of the growth annuity due paid over t years:

$$Annuity\ FV(t, r, x, g) = \frac{(1+r)^t - (1+g)^t}{r-g}\, x \qquad \text{when } r \neq g$$

$$Annuity\ FV(t, r, x, g) = t(1 + r)^{t-1} x \qquad \text{when } r = g$$

$$Annuity\ Due\ FV(t, r, x, g) = (1 + r) * Annuity\ FV(t, r, x, g)$$

Applying the ordinary growth annuity formula to the current problem yields the following answer:

$$Annuity\ FV(t = 4, r = 3\%, x = \$1{,}000, g = 20\%) = \frac{(1+0.03)^4 - (1+0.2)^4}{0.03-0.2} * \$1{,}000 = \$5{,}577.01$$

A7b. The future value of the ordinary annuity and the sum of future values of all annuity payments (i.e. deposits made to saving account in problem 6) are the same. An annuity is a series of payments made at equal intervals, a.k.a. periods. In the ordinary annuity the payment is made at the and of the period. The four deposits made on 31 December over the four consecutive years constitute an ordinary annuity.

Thus, the future value of the ordinary annuity on 31 December 2022 can be calculated by summing up future values of all four deposits

$$Annuity\ FV(t=4,r,x,g) = (1+r)^3\,(1+g)^0x + (1+r)^2\,(1+g)^1x + (1+r)^1\,(1+g)^2x + (1+r)^0\,(1+g)^3x$$

$$Annuity\ FV(t=4,r,x,g) = (1+0.03)^3\,(1+0.2)^0 * \$1{,}000 + (1+0.03)^2\,(1+g)^1 * \$1{,}000 + (1+0.03)^1\,(1+0.2)^2 * \$1{,}000 + (1+r)^0\,(1+g)^3 * \$1{,}000$$

$$Annuity\ FV(t=4,r=3\%,x=\$1{,}000,g=20\%) = \$1{,}092.73 + \$1{,}273.08 + \$1{,}483.20 + \$1{,}728 = \$5{,}577.01$$

A7c. Derive formula for ordinary annuity with the annuity payments growing at rate g annually.

Start from the equation shown in point b that represents sum of the future values of all deposits, namely:

$$Annuity\ FV(t=4,r,x,g) = (1+r)^3\,(1+g)^0x + (1+r)^2\,(1+g)^1x + (1+r)^1\,(1+g)^2x + (1+r)^0\,(1+g)^3x$$

Generalize the above equation for any arbitrary value of t, namely:

$$\begin{aligned}\boldsymbol{Annuity\ FV(t,r,x,g)} &= \boldsymbol{(1+r)^{t-1}\,(1+g)^{t-t}x + (1+r)^{t-2}\,(1+g)^{t-(t-1)}x} \\ &\quad \boldsymbol{+ (1+r)^{t-3}\,(1+g)^{t-(t-2)}x + \cdots + (1+r)^{t-(t+1)}\,(1+g)^{t-(t-t+1)}x} \\ &\quad \boldsymbol{+ (1+r)^{t-t}\,(1+g)^{t-(t-t+1)}x}\end{aligned}$$

Further simplify the above equation

$$Annuity\ FV(t,r,x,g) = \left[(1+r)^{t-1}\,(1+g)^0 + (1+r)^{t-2}\,(1+g)^{t-(t-1)} + (1+r)^{t-3}\,(1+g)^{t-(t-2)} + \cdots + (1+r)^1\,(1+g)^{t-2} + (1+r)^0\,(1+g)^{t-1}\right]x \qquad \text{(i)}$$

When $r \neq g$

Multiply the above equation (i) by (1+r) / (1+g)

$$Annuity\ FV(t,r,x,g)\,\frac{1+r}{1+g} = \left[(1+r)^t\,(1+g)^{-1} + (1+r)^{t-1}\,(1+g)^0 + (1+r)^{t-2}\,(1+g)^{t-(t-1)} + \cdots + (1+r)^2\,(1+g)^{t-3} + (1+r)^1\,(1+g)^{t-2}\right]x \qquad \text{(ii)}$$

Subtract equation (i) from equation (ii), namely:

$$Annuity\ FV(t,r,x,g)\left(\frac{1+r}{1+g}-1\right)=[(1+r)^t\,(1+g)^{-1}-\,(1+r)^0\,(1+g)^{t-1}]x$$

(iii)

Multiply both sides of the above (iii) by (1+g), namely:

$$Annuity\ FV(t,r,x,g)\,(\,1+r-1-g)=[(1+r)^t\,(1+g)^{-1+1}-\,(1+r)^0\,(1+g)^{t-1+1}]x$$

$$Annuity\ FV(t,r,x,g)\,(\,r-g)=[(1+r)^t\,-\,(1+g)^t]x$$

(iv)

Divide both sides of equation (iv) by (r - g) to derive future value growth annuity formula

$$Annuity\ FV(t,r,x,g\neq r)\ =\frac{(1+r)^t-\,(1+g)^t}{r-g}\,x$$

When $r=\,g$

Substitute g for r in equation (i), namely:

$$\begin{aligned}Annuity\ FV(t,r,x,g=r)&\\ &=\big[(1+r)^{t-1}\,(1+r)^0+(1+r)^{t-2}\,(1+r)^{t-(t-1)}\\ &+(1+r)^{t-3}\,(1+r)^{t-(t-2)}+\cdots+(1+r)^1\,(1+r)^{t-2}\\ &+(1+r)^0\,(1+r)^{t-1}\big]x\end{aligned}$$

Simplify the above equation, namely:

$$\begin{aligned}Annuity\ FV(t,r,x,g=r)&\\ &=[(1+r)^{t-1}+(1+r)^{t-1}+(1+r)^{t-1}+\cdots+(1+r)^{t-1}\\ &+(1+r)^{t-1}]x\end{aligned}$$

Note that term $(1+r)^{t-1}$ occurs exactly t times, and thus, equation simplifies to:

$$Annuity\ FV(t,r,x,g=r)=t(1+r)^{t-1}x$$

A8.

$$Annuity\ FV(t,r,n,x) = \left(\frac{-1+\left(1+\frac{r}{n}\right)^{nt}}{\frac{r}{n}}\right)x$$

$$Annuity\ FV(t=5, r=3\%, n=12, \mathrm{x}=\$500) = \left(\frac{\left(1+\frac{0.03}{12}\right)^{12*5}-1}{\frac{0.03}{12}}\right) * 500 = \$290{,}744.79$$

A9a. Calculate weekly equivalent nominal rate

Compounding	Periods per Year	Nominal Rate	Effective Rate
Monthly	12	2.5%	2.52885% calculated as $-1+\left(1+\frac{0.025}{12}\right)^{12}$
Weekly	52	2.49799% calculated as $52 * \left[-1+\left(1+\frac{0.025}{12}\right)^{\frac{12}{52}}\right]$	2.52885% Must be the same as the monthly effective rate to calculate equivalent nominal rate

$$\left(1+\frac{r_{weekly}}{52}\right)^{52} = \left(1+\frac{0.025}{12}\right)^{12}$$

$$r_{weekly} = 52 * \left[-1+\left(1+\frac{0.025}{12}\right)^{\frac{12}{52}}\right] = 2.49799\%$$

A9b. Calculate the future value of the ordinary annuity

$$Annuity\ FV(t,r,\mathrm{n},x) = \left(\frac{-1+\left(1+\frac{r}{n}\right)^{nt}}{\frac{r}{n}}\right)x$$

$$Annuity\ FV(t=35, r=2.49799\%, n=52, \mathrm{x}=\$100) = \left(\frac{\left(1+\frac{0.0249799}{52}\right)^{52*35}-1}{\frac{0.0249799}{52}}\right) *100 =$$

$290,744.79

A10a. Calculate the future value of the initial investment

$$FV(t,r,n,x) = \left(1+\frac{r}{n}\right)^{nt} x$$

$$FV(t=10, r=5\%, n=2, x=\$600) = \left(1+\frac{0.05}{2}\right)^{2*10} * 600 = \mathbf{983.17}$$

A10b. Calculate 2-month equivalent nominal rate

Compounding	Periods per Year	Nominal Rate	Effective Rate
Semi-annual	2	5%	5.0625% calculated as $-1+\left(1+\frac{0.05}{2}\right)^2$
2-month period	6	4.9589% calculated as $6*\left[-1+\left(1+\frac{0.05}{2}\right)^{\frac{2}{6}}\right]$	5.0625% Must be the same as the semi-annual effective rate to calculate equivalent nominal rate

$$\left(1+\frac{r_{2\_mont}}{6}\right)^6 = \left(1+\frac{0.05}{12}\right)^{12}$$

$$r_{2\_mont} = 6*\left[-1+\left(1+\frac{0.05}{2}\right)^{\frac{2}{6}}\right] = 4.9589\%$$

A10c. Calculate the future value of the ordinary annuity

$$Annuity\ FV(t,r,n,x) = \left(\frac{-1+\left(1+\frac{r}{n}\right)^{nt}}{\frac{r}{n}}\right)x$$

$$Annuity\ FV(t=10, r=4.9589\%, n=6, \mathrm{x}=\$200) = \left(\frac{\left(1+\frac{0.049589}{6}\right)^{6*10}-1}{\frac{0.049589}{6}}\right) * 200 =$$

\$15,453.82

A10d. Calculate the future value of the mutual fund holdings

$$FV\ of\ mutual\ fund\ holdings = FV(t=10, r=5\%, n=2, x=\$600) + Annuity\ FV(t=10, r=4.9589\%, n=6, \mathrm{x}=\$200)$$

$$FV\ of\ mutual\ fund\ holdings = 983.17 + \$15{,}453.82 = \$16{,}436.99$$

P11a. Calculate monthly equivalent nominal rate

Compounding	Periods per Year	Nominal Rate	Effective Rate
Quarterly	4	4%	4.06040% calculated as $-1+\left(1+\frac{0.04}{4}\right)^{4}$
Monthly	12	3.98674% calculated as $12 * \left[-1+\left(1+\frac{0.05}{2}\right)^{\frac{4}{12}}\right]$	4.06040% Must be the same as the monthly effective rate to calculate equivalent nominal rate

$$\left(1+\frac{r_{monthly}}{12}\right)^{12} = \left(1+\frac{0.04}{4}\right)^{4}$$

$$r_{monthly} = 12 * \left[-1+\left(1+\frac{0.05}{2}\right)^{\frac{4}{12}}\right] = 3.98674\%$$

P11b. Calculate monthly deposits

$$Annuity\ FV(t,r,n,x) = \left(\frac{-1+\left(1+\frac{r}{n}\right)^{nt}}{\frac{r}{n}}\right)x$$

$$Annuity\ FV(t=6, r=3.98674\%, n=12, \mathrm{x}) = \left(\frac{\left(1+\frac{0.0398674}{12}\right)^{12*6}-1}{\frac{0.0398674}{12}}\right) \mathrm{x} = 66.2767\ \mathrm{x}$$

$$Annuity\ FV(t=6, r=3.98674\%, n=12, \mathrm{x}) = \$5{,}000$$

66.2767 x = $5,000

x = $5,000 / 66.2767 = $75.44

A12a. Calculate the 4-month period equivalent nominal rate.

Compounding	Periods per Year	Nominal Rate	Effective Rate
Monthly	12	3%	3.04160% calculated as $-1+\left(1+\frac{0.03}{12}\right)^{12}$
4-month period	3	3.01127% calculated as $12 * \left[-1+\left(1+\frac{0.05}{2}\right)^{\frac{4}{12}}\right]$	3.04160% Must be the same as the monthly effective rate to calculate equivalent nominal rate

$$\left(1+\frac{r_{4\_month}}{3}\right)^{3} = \left(1+\frac{0.03}{12}\right)^{12}$$

$$r_{4\_mont} = 3 * \left[-1+\left(1+\frac{0.03}{12}\right)^{\frac{12}{3}}\right] = 3.01127\%$$

A12b. Calculate the present value of the ordinary annuity

$$Annuity\ PV(t,r,n,x) = \left(\frac{1-\left(1+\frac{r}{n}\right)^{-n}}{\frac{r}{n}}\right)x$$

$$Annuity\ PV(t=5, r=3\%, n=3, x=\$200) = \left(\frac{1-\left(1+\frac{0.0301127}{3}\right)^{-3*5}}{\frac{0.0301127}{3}}\right) * 200$$

$$= \$2{,}772.20$$

A12c. Calculate the total interest paid

$$total\ interest = n*t*x - Annuity\ PV = 3*5*200 - 2{,}772.20 = \$227.80$$

A13a. Calculate 4-month period equivalent nominal rate

Compounding	Periods per Year	Nominal Rate	Effective Rate
Quarterly	4	6%	6.13636% calculated as $-1+\left(1+\frac{0.06}{4}\right)^{4}$
Monthly	12	5.97025% calculated as $12 * \left[-1+\left(1+\frac{0.05}{2}\right)^{\frac{4}{12}}\right]$	6.13636% Must be the same as the quarterly effective rate to calculate equivalent nominal rate

$$\left(1+\frac{r_{monthly}}{12}\right)^{12} = \left(1+\frac{0.06}{4}\right)^{4}$$

$$r_{monthly} = 12 * \left[-1+\left(1+\frac{0.06}{4}\right)^{\frac{4}{12}}\right] = 5.97025\%$$

A13b. Calculate the monthly payment required on the first day of the month

$$Annuity\ Due\ PV(t,r,n,x) = (1+\frac{r}{n})\left(\frac{1-\left(1+\frac{r}{n}\right)^{-n}}{\frac{r}{n}}\right)x$$

$$Annuity\ Due\ PV(t=9, r=\ 5.97025\%\ , n=12, x)$$
$$= (1+\frac{0.0597025}{12})\left(\frac{1-\left(1+\frac{0.0597025}{12}\right)^{-1\ *9}}{\frac{0.0597025}{12}}\right)x$$

$$Annuity\ Due\ PV(t=9, r=\ 5.97025\%\ , n=12, x) = 83.81050\ x$$

$$Annuity\ Due\ PV(t=9, r=\ 5.97025\%\ , n=12, x) = \$80{,}000 - \$15{,}500 = \$64{,}500$$

$$83.81050\ \ x = \$64{,}500$$

$$x = \frac{\$64{,}500}{83.81050} = \ \$769.59$$

A13c. Calculate total interest paid

$$total\ interest = n * t * x - Annuity\ Due\ PV(t,r,n,x)$$
$$total\ interest = 12 * 9 * 769.59 - 64500 =\ 83{,}116.08\ - 64500$$
$$total\ interest = \$18{,}616.08$$

A14a. Calculate 4-month period equivalent nominal rate

Compounding	Periods per Year	Nominal Rate	Effective Rate
Quarterly	4	6%	6.13636% calculated as $-1+\left(1+\frac{0.06}{4}\right)^4$
Monthly	12	5.97025% calculated as $12 * \left[-1+\left(1+\frac{0.05}{2}\right)^{\frac{4}{12}}\right]$	6.13636% Must be the same as the quarterly effective rate to calculate equivalent nominal rate

$$\left(1+\frac{r_{month}}{12}\right)^{12} = \left(1+\frac{0.06}{4}\right)^4$$

$$r_{monthly} = 12 * \left[-1+\left(1+\frac{0.06}{4}\right)^{\frac{4}{12}}\right] = 5.97025\%$$

A14b. Calculate the monthly payment required on the last day of the month

$$Annuity\ PV(t,r,n,x) = \left(\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right)x$$

$$Annuity\ PV(t=9, r= 5.97025\%, n=12, x)$$
$$= (1+\frac{0.0597025}{12})\left(\frac{1-\left(1+\frac{0.0597025}{12}\right)^{-12*9}}{\frac{0.0597025}{12}}\right)x$$

$$Annuity\ PV(t=9, r= 5.97025\%, n=12, x) = 83.39559\,x$$

$$Annuity\ PV(t=9, r= 5.97025\%, n=12, x) = \$80{,}000 - \$15{,}500 = \$64{,}500$$

$$83.39559\ x = \$64{,}500$$

$$x = \frac{\$64{,}500}{83.39559} = \ \$773.42$$

A14c. Calculate total interest paid

$$total\ interest = n * t * x - Annuity\ PV(t,r,n,x)$$
$$total\ interest = 12 * 9 * \ \$773.42 - 64500 = \ \$83{,}529.60\ - 64500$$
$$total\ interest = \ \$19{,}029.60$$

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